Homework based on Chapter 6
Computational Probability and Statistics
CIS 2033, Section 002

1 Part 1 (Due: 9:00 AM, Friday, Feb. 20, 2015)

Question 1 Let U follows a $U(0,1)$ distribution. Let X follows a binomial distribution $Bin(3, 0.5)$.Describe how to simulate the outcome of X. (Hint: X can take values from $k = \{0, 1, 2, 3\}$, $p(k) = \binom{3}{k}(0.5)^k(0.5)^{3-k}$)

Question 2 Somebody messed up the random number generator in your computer, instead of uniform random numbers it generates numbers with an $Par(3)$ distribution. Describe how to construct a $U(0,1)$ random variable U from an $Par(3)$ distributed X.

2 Part 2 (Due: 11:59 PM, Tuesday, 24, 2015)

Question 3 A random variable X has a $Exp(2)$ distribution. For details of the Exponential distribution see the notes. Describe how to construct X from $U(0,1)$ random variable.

Question 3 A random variable X has a $Par(5)$ distribution. For details of the Exponential distribution see the notes. Describe how to construct X from $U(0,1)$ random variable.

Sample Questions

6.1 Let U have a $U(0,1)$ distribution. Describe how to simulate the outcome of a roll with a die using U.

Answer. Denote X the random variable for the outcome of throwing a die. X can take $k = 1, 2, 3, 4, 5, 6$. The PMF of X is $P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6) = \frac{1}{6}; 0$ elsewhere. To simulate the PMF of X, is to define a partition on Ω of U such that the area of each part corresponds to a probability of X.

Table 1: Simulation of rolling a die using $U(0,1)$.

Events of U	$0 \leq U < \frac{1}{6}$	$\frac{1}{6} \leq U < \frac{2}{6}$	$\frac{2}{6} \leq U < \frac{3}{6}$	$\frac{3}{6} \leq U < \frac{4}{6}$	$\frac{4}{6} \leq U < \frac{5}{6}$	$\frac{5}{6} \leq U \leq 1$
Events of X	X=1	X=2	X=3	X=4	X=5	X=6
Equal probabilities	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Notice: the second row of the table can be shuffled, which is also a valid simulation.

6.6 Somebody messed up the random number generator in your computer, instead of uniform random numbers it generates numbers with an $Exp(2)$ distribution. Describe how to construct a $U(0,1)$ random variable U from an $Exp(2)$ distributed X.

Answer.For the uniform distribution, $U(0,1)$, the distribution function is $F_U(u) = u$ for $0 \leq u \leq 1$. For the exponential distribution, $Exp(2)$, the distribution function is $F_X(x) = 1 - e^{-2x}$ for $x \geq 0$. In order to construct a $U(0,1)$ random variable U from an $Exp(2)$ distributed X, we have to compute $F_U(u) = F_X(x)$. This means $u = 1 - e^{-2x}$ for $x \geq 0$. Then, we can define the $U(0,1)$ distributed random variable $U = F^{inv}(X) = 1 - e^{-2X}$. If U is a uniform distribution, then $1 - U$ is also a uniform distribution, we can also construct $U = e^{-2X}$.