

$$\begin{aligned} 3. \quad \text{area of a triangle} &= (\text{height} * \text{base}) / 2 \\ &= (\text{height} * 1) / 2 & \text{base} = 1 \\ &= \text{height} / 2 \end{aligned}$$

$$\begin{aligned} \text{a) } \max \text{ height} &= 1 \\ \max \text{ area} &= 1/2 \end{aligned}$$

$$\text{height} / 2 \leq 1/4$$

$$\text{height} \leq 1/2$$

$$\text{for } A \leq 1/4, \{ (x, y) \mid 2 \leq x \leq 3, 1 \leq y \leq 1.5 \}$$

b) All triangle heights from 0 to 1 are possible. Since the point is arbitrarily chosen (all points equally probable), all areas from 0 to 1/2 are equally probable.

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 2x & \text{for } 0 \leq x \leq 1/2 \\ 1 & \text{for } x > 1/2 \end{cases}$$

$$\text{c) } f(x) = d/dx F(x)$$

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 2 & \text{for } 0 \leq x \leq 1/2 \\ 0 & \text{for } x > 1/2 \end{cases}$$

$$4. \quad \text{Exp}(3)$$

$$\begin{aligned} F(a) &= 1 - e^{-\lambda a} \\ &= 1 - e^{-3a} \end{aligned}$$

$$0.5 = 1 - e^{-3a}$$

$$-0.5 = -e^{-3a}$$

$$0.5 = e^{-3a}$$

$$\ln(0.5) = \ln(e^{-3a})$$

$$\ln(0.5) = -3a$$

$$a = \ln(0.5) / -3$$

$$a = 0.2310$$

The median of an Exp(3) distribution is 0.2310.

$$\begin{aligned}
 5. \quad & \text{Par}(1) \\
 & F(x) = 1 - 1/x^\alpha \\
 & \quad = 1 - 1/x^1 = 1 - 1/x
 \end{aligned}$$

$$\begin{aligned}
 0.5 &= 1 - 1/x \\
 -0.5 &= -1/x \\
 0.5 &= 1/x \\
 0.5x &= 1 \\
 x &= 2
 \end{aligned}$$

The median of a Par(1) distribution is 2.