4

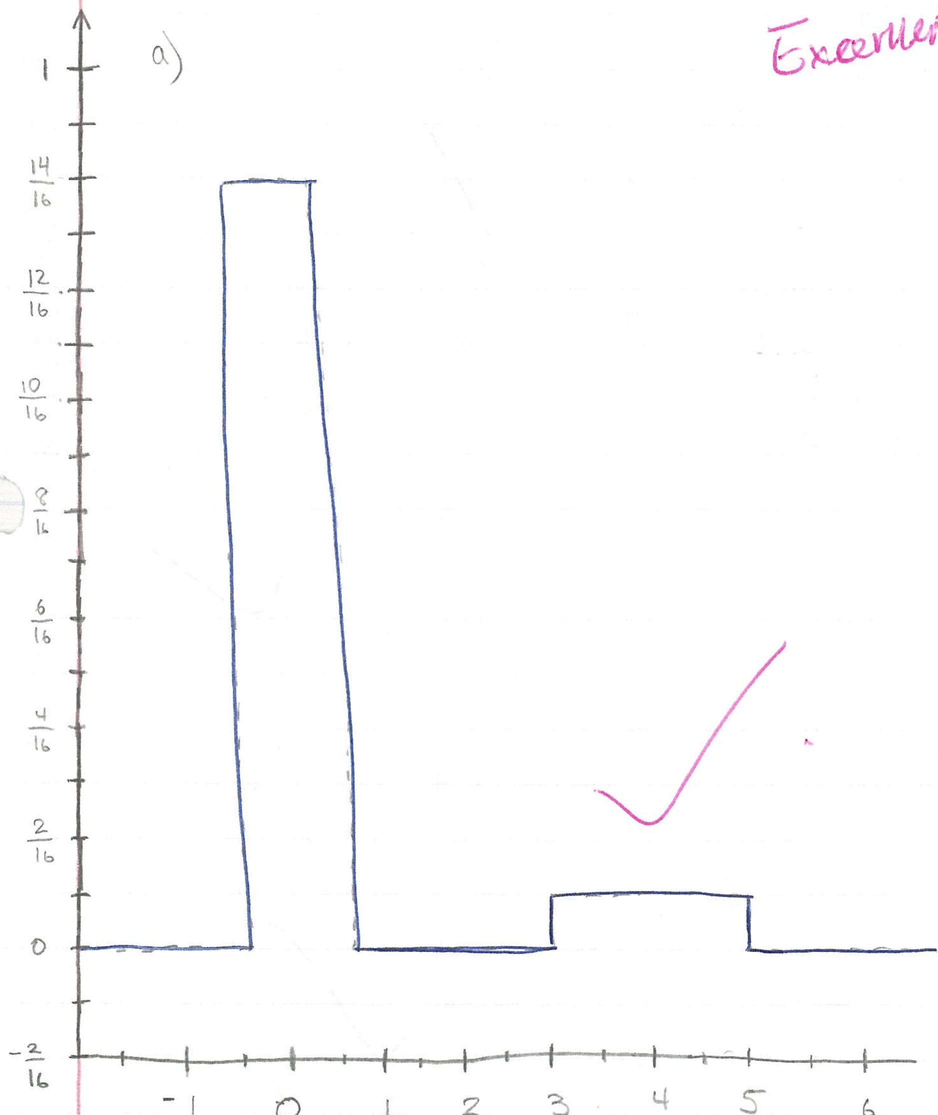
[redacted] - HW Based on Chapter 5 2/13/15

Part 1

Question 1 :

a)

Excellent



b) If $a < -\frac{1}{2}$ $\quad F(a) = \int_{-\infty}^{a} 0\,dx = 0$

If $-\frac{1}{2} \leq a < \frac{1}{2}$ $\quad F(a) = \int_{-\infty}^{0} 0\,dx + \int_{-\frac{1}{2}}^{a} \frac{7}{8}\,dx$

$$0 + \left[\frac{7}{8}x\right]_{-\frac{1}{2}}^{a}$$

$$0 - \frac{7}{16} + \frac{7}{8}a$$

If $1 \leq a < 3$ $\quad F(a) = \int_{-\infty}^{0} 0\,dx + \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{7}{8}\,dx + \int_{1}^{a} 0\,dx$

$$0 + \frac{7}{8}x\Big]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$0 + \frac{7}{8} + 0 = \frac{7}{8}$$

If $3 \leq a < 5$ $\quad F(a) = \int_{-\infty}^{0} 0\,dx + \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{7}{8}\,dx + \int_{1}^{3} 0\,dx + \int_{3}^{a} \frac{1}{16}\,dx$

$$= 0 + \frac{7}{8} + 0 + \frac{1}{16}x\Big]_{3}^{a}$$

$$= 0 + \frac{7}{8} + 0 + \left(\frac{a}{16} - \frac{3}{16}\right)$$

$$= \frac{14}{16} + \left(\frac{a-3}{16}\right) = \frac{11+a}{16}$$

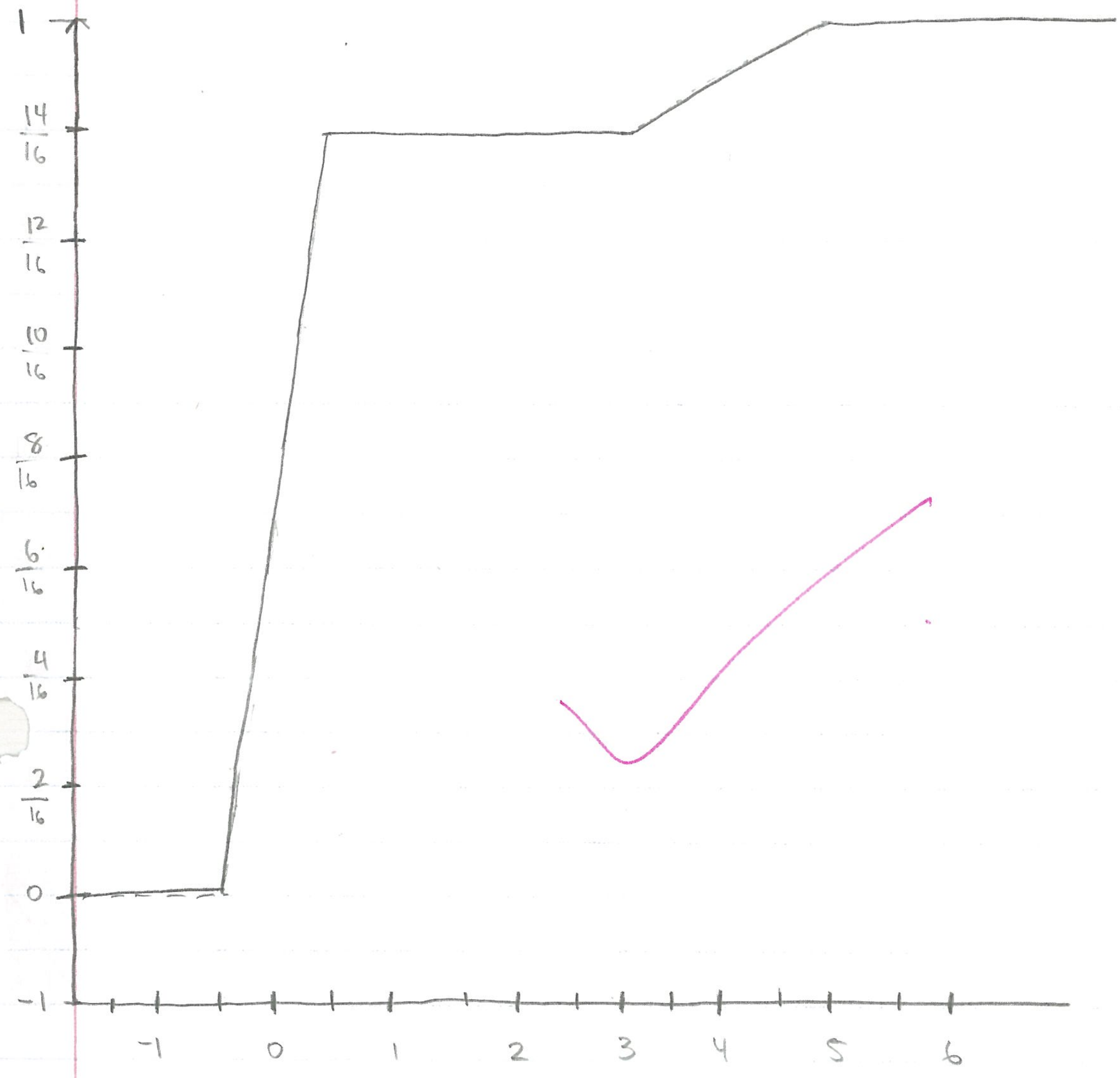
If $5 \leq a$

$$F(a) = \int_{-\infty}^{0} 0\,dx + \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{7}{8} + \int_{1}^{3} 0\,dx + \int_{3}^{5} \frac{1}{16}\,dx + \int_{5}^{a} 0\,dx$$

$$0 + \frac{7}{8} + 0 + \left(\frac{5}{16} - \frac{3}{16}\right) + 0$$

$$= 0 + \frac{7}{8} + 0 + \frac{2}{16} + 0$$

$$= \frac{14}{16} + \frac{2}{16} = \frac{16}{16} = \boxed{1}$$



Question 2 :

a) Compute c

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \qquad \int_0^2 c\,(4x - 2x^2)\,dx = 1$$

$$c\left(2x^2 - \frac{2}{3}x^3\right)\Big]_0^2 = 1 \qquad c\left(\frac{24}{3} - \frac{16}{3}\right) = 1 \qquad \boxed{c = \frac{3}{8}}$$

$$c\,\frac{8}{3} = 1 \rightarrow \frac{3}{8}$$

b. $F(x) = \int_0^x c(4x - 2x^2)\,dx$

$= c\left(2x^2 - \frac{2}{3}x^3\right)\Big]_0^x$

We plug in c from part A.

$= \frac{3}{8}\left(2x^2 - \frac{2}{3}x^3\right)$

Probability Density Function

$$F(x) = \begin{cases} \frac{3}{8}\left(2x^2 - \frac{2}{3}x^3\right) & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere.} \end{cases}$$

c. $F(1) = \int_1^2 c(4x - 2x^2) = c\left(2x^2 - \frac{2}{3}x^3\right)\Big]_1^2$

$= c\left(\frac{24}{3} - \frac{16}{3}\right) - \left(\frac{6}{3} - \frac{2}{3}\right) = c\left(\frac{8}{3} - \frac{4}{3}\right) = c\,\frac{4}{3} = \boxed{\frac{1}{2}}$

$P(X > 1) = \int_1^2 c(4x - 2x^2) = c\left(2x^2 - \frac{2}{3}x^3\right)\Big]_1^2$

$= c\left(\left(\frac{24}{3} - \frac{16}{3}\right) - \left(\frac{6}{3} - \frac{2}{3}\right)\right)$

$c\left(\frac{8}{3} - \frac{4}{3}\right) = c\,\frac{4}{3} = \frac{3}{8} \cdot \frac{4}{3} \quad \frac{12}{24} = \boxed{\frac{1}{2}}$