Revise(Chapter 5): CIS 2033 Sprint 2015: Computational Prob and Stat
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1 Notations

X, Y, Z: Random Variables.
k, a, b: specific whole number (e.g. 0,1,2,...).
x, y: specific real number (e.g. 1.2, 0.5, ...).
$p(k)$: Probability mass function for discrete random variable X. It calculate for any specific whole number k, the probability of $P(X = k)$.
$f(x)$: Probability density function for continuous random variable X.
$F(x) = P(X \leq x)$: Distribution function, sometimes it's called cumulative distribution function.

If X is discrete: $F(k) = \sum_{y \leq k} p(y)$, where y is any possible value for X that is less or equal than k.

If X is continuous: $F(x) = \int_{-\infty}^{x} f(x)dx$

1.1 Important distributions

	Notation	$p(k)$ **or** $f(x)$	$F(k)$ **or** $F(x)$
Discrete	$X \sim Ber(p)$	$p(1) = p; p(0) = 1 - p$	$F(k) = 0, (k < 0); F(k) = 1 - p, (0 \leq k < 1); F(k) = 1, (k > 1)$
	$X \sim Bin(n, p)$	$p(k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, 1, ..., n$	$F(k) = \sum_{y \leq k} p(y), k = 0, 1, ..., n$
	$X \sim Geo(p)$	$p(k) = (1-p)^{k-1} p, k = 1, ...$	$F(k) = 1 - (1-p)^k, k = 0, 1, ...$
	$X \sim Pois(\mu)$	$p(k) = \frac{\mu^k}{k!} e^{-k}, k = 0, 1, ...$	$F(k) = \sum_{y \leq k} p(y), k = 0, 1, ...$
Continuous	$X \sim Unif(\alpha, \beta)$	$f(x) = \frac{1}{\beta - \alpha}, for\ x \in (\alpha, \beta)$	$F(x) = \frac{x - \alpha}{\beta - \alpha}, for\ x \in (\alpha, \beta)$
	$X \sim Exp(\lambda)$	$f(x) = e^{-\lambda x}, for\ x \in [0, \infty)$	$F(x) = 1 - e^{-\lambda x}, for\ x \in [0, \infty)$
	$X \sim Par(\alpha)$	$f(x) = \frac{\alpha}{x^{\alpha+1}}, for\ x \in [1, \infty)$	$F(x) = 1 - \frac{1}{x^\alpha}, for\ x \in [1, \infty)$
	$X \sim N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in (-\infty, +\infty)$	NO explicit form

2 Table of Basic Integrals

2.1 Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1},\ \ n \neq -1 \tag{1}$$

$$\int \frac{1}{x} dx = \ln |x| \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| \tag{4}$$

2.2 Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x},\ \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$