Homework based on Chapter 4
Computational Probability and Statistics
CIS 2033, Section 002

1 Part 1 (Due: 9:00 AM, Friday, Feb. 06, 2015)

Question 1 Let X be a discrete random variable with Probability Mass Function p is given by:

a	-1	0	1	2
$p(a)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$

and $p(a)$=0 for all other a.

a. Let the random variable Y be defined by $Y = X^2$, i.e., if X = 2, then Y = 4. Calculate the probability mass function of Y.

b. Calculate the value of the distribution function of X and Y in a = 1, a = $\frac{3}{4}$, and $a = \pi - 3$.

Question 2 Suppose that the distribution function of a discrete random variable X is given by

$$F(a) = \begin{cases} 0 & \text{for} \quad a < 0 \\ \frac{1}{3} & \text{for} \quad 0 \leq a < \frac{1}{2} \\ \frac{2}{3} & \text{for} \quad \frac{1}{2} \leq a < \frac{3}{4} \\ 1 & \text{for} \quad a \geq \frac{3}{4}. \end{cases}$$

Determine the probability mass function of X.

2 Part 2 (Due: 05:30 PM, Tuesday, Feb. 10, 2015)

Question 1 Given a dice with six numbers ($\{1, 2, 3, 4, 5, 6\}$), each number comes with the same probability when you roll it. Here is the game. Suppose you have such TWO dices and you simultaneously roll both of them to get the product of the two output numbers. When the product is 1 or 36, we say that you get the magic numbers and you will be rewarded. However, each play will cost you a certain amount of money and you can only afford to play 100 times. Let the random variable X denote the total number of times you will hit those magic numbers and be rewarded. Answer the following questions:

a. What is the distribution of X? Specify its parameters.

b. What is the probability mass function of X?

Question 2 Given a dice with six numbers ($\{1, 2, 3, 4, 5, 6\}$), each number comes with the same probability when you roll it. Here is the game. Suppose you have such TWO dices and you simultaneously roll both of them to get the product of the two output numbers. When the product is 1 or 36, we say that you get the magic numbers and you will be rewarded. However you can play it as many times as possible until you win, then you stop. Let the random variable X denote the total number of times before you stop. Answer the following questions:

a. What is the distribution of X? Specify its parameters.

b. What is the probability mass function of X?

3 Part 3 Extra Credits 10 (Due: 05:30 PM, Tuesday, Feb. 10, 2015)

Question 3 You toss n coins, each showing heads with probability p, independently of the other tosses. Each coin that shows tails is tossed again once. Let X be the total number of heads.

a. What type of distribution does X have? Specify its parameter(s).

b. What is the probability mass function of the total number of heads X?

Question 4 You decide to play monthly in two different lotteries, and you stop playing as soon as you win a prize in one (or both) lotteries of at least one million dollars. Suppose that every time you participate in these lotteries, the probability to win at least one million dollars is p_1 for one of the lotteries and p_2 for the other. Let M be the number of times you participate in these lotteries until winning at least one prize. What kind of distribution does M have, and what is its parameter?

Question 5 We throw a coin until a head turns up for the second time, where p is the probability that a throw results in a head and we assume that the outcome of each throw is independent of the previous outcomes. Let X be the number of times we have thrown the coin.

a. Determine $P(X = 2), P(X = 3)$, and $P(X = 4)$.

b. Show that $P(X = n) = (n - 1)p^2(1 - p)^{n-2}$, for $n \geq 2$.

Appendix

Sample Q1 Let X be a discrete random variable with Probability Mass Function p is given by:

a	-1	0	1	2
$p(a)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

and $p(a)$=0 for all other a.

a. Let the random variable Y be defined by $Y = X^2$, i.e., if X = 2, then Y = 4. Calculate the probability mass function of Y.

b. Calculate the value of the distribution function of X and Y in a = 1, a = $\frac{3}{4}$, and $a = \pi - 3$.

Answer a) The *probability mass function* of a discrete random variable X is the function $p : \mathbb{R} \to [0, 1]$, defined by

$$p(a) = P(X = a), \text{ for} - \infty < a < \infty$$

X	-1	0	1	2
$Y = X^2$	1	0	1	4
prob.	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

Table 1: The probability mass function of X

Make sure the uniqueness of sample space for Y random variable, Y can take values from $\{0, 1, 4\}$, for $P(Y =)$we get Table 2.

b	0	1	4
$p(b)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$

Table 2: The probability mass function of Y

Answer b) *The distribution function F* of a random variable X is the function F: $\mathbb{R} \to [0, 1]$, defined by

$$F(a) = P(X \leq a), \text{ for} - \infty < a < \infty$$

a	-1	0	1	2
$p(a)$.	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

Table 3: The probability mass function of X

For X,

$$\begin{aligned}
F(X=1) &= P(X \leq 1) \\
&= P(X=-1) + P(X=0) + P(X=1) \\
&= 0.25 + 0.125 + 0.125 \\
&= 0.5 \\
F(X=\frac{3}{4}) &= P(X \leq 0.75) \\
&= P(X=-1) + P(X=0) \\
&= 0.25 + 0.125 \\
&= 0.375 \\
F(X=\pi-3) &= P(X \leq \pi-3) \\
&= P(X=-1) + P(X=0) \\
&= 0.25 + 0.125 \\
&= 0.375
\end{aligned}$$

For Y, $a = 1, \frac{3}{4}, \pi - 3$ means $b = 1, (\frac{3}{4})^2, (\pi-3)^2$

b	0	1	4
$p(b)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$

Table 4: The probability mass function of Y

$$
\begin{aligned}
F(Y = 1) &= P(Y \leq 1) \\
&= P(Y = 0) + P(Y = 1) \\
&= 0.125 + 0.375 \\
&= 0.5 \\
F(Y = \frac{9}{16}) &= P(Y \leq \frac{9}{16}) \\
&= P(Y = 0) \\
&= 0.125 \\
F(Y = (\pi - 3)^2) &= P(Y \leq (\pi - 3)^2) \\
&= P(Y = 0) \\
&= 0.125
\end{aligned}
$$