Homework based on Chapter 3
Computational Probability and Statistics
CIS 2033, Section 002

Due: 9:00 AM, Friday, Jan. 30, 2015

Question 1 A certain grapefruit variety is grown in two regions in southern Spain. Both areas get infested from time to time with parasites that damage the crop. Let A be the event that region R 1 is infested with parasites and B that region R_2 is infested. Suppose $P(A) = 1/4, P(B) = 1/2$ and $P(A \cup B) = 5/8$. If the food inspection detects the parasite in a ship carrying grapefruits from R_1 , what is the probability region R_2 is infested as well?

Question 2 A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy $P(I \cap E) = P(I^c \cap E^c) = p$, where I is the event "breath analyzer indicates that legal limit is exceeded" and E "drivers blood alcohol percentage exceeds legal limit." On Saturday night about 7% of the drivers are known to exceed the limit.

(a) Describe in words the meaning of $P(E^c|I)$.

(b) Determine $P(E^c|I)$ if $p = 0.90$.

(c) How big should p be so that $P(E|I) = 0.90$?

Question 3 At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however, that a new piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect has the characteristic?

Question 4 You are diagnosed with an uncommon disease. You know that there only is a 1% chance of getting it. Use the letter D for the event "you have the disease" and T for "the test says so." It is known that the test is imperfect: $P(T|D) = 0.95$ and $P(T^c|D^c) = 0.90$.

(a) Given that you test positive, what is the probability that you really have the disease?

(b) You obtain a second opinion: an independent repetition of the test. You test positive again. Given this, what is the probability that you really have the disease?

Question 5 Suppose A and B are events with $0 < P(A) < 1$ and $0 < P(B) < 1$.

(a) If A and B are disjoint, can they be independent?

(b) If A and $B$are independent, can they be disjoint?

(c) If $A \in B$, can A and B be independent?

(d) If A and B are independent, can A and $A \cup B$ be independent?