Computational Probability and Statistics

Homework 1 01/20/2015

[redacted]

Chapter 2: Outcomes, events and probability

Question 1:

Let E and F be two events in a sample space for which P(E)=$\frac{1}{3}$ and P(F)=$\frac{1}{2}$ and P(E∪F)=$\frac{2}{3}$, what is P(E∩F)?

P(E∪F)=P(E)+P(F)−P(E∩F)

⇒P(E∩F)=P(E)+P(F)−P(E∪F)=$\frac{1}{3}+\frac{1}{2}-\frac{2}{3}=\frac{1}{6}$

Question 2:

Let A and B be two events for which one knows that the probability that at least one of them occurs is $\frac{2033}{3302}$. What is the probability that neither A nor B occurs?

Hint: use one of DeMorgan's laws: $A^c \cap B^c = (A \cup B)^c$

From DeMorgan's laws,

the probability that neither A nor B occurs is

$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - \frac{2033}{3302} = \frac{1269}{3302}$

Question 3:

We consider events A, B and C, which can occur in some experiment. Is it true that the probability that only A occurs and not B or C) is equal to P(A∪B∪C)−P(B)−P(C)+P(B∩C)?

Yes, it is true.

Pf: $P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(A \cap B)-P(A \cap C)-P(B \cap C)+P(A \cap B \cap C)$ (*)

The probability that only A occurs is

$P(A \cap B^c \cap C^c) = P(A \cap (B \cup C)^c) = P(A) - P(A \cap (B \cup C))$

$= P(A) - P((A \cap B) \cup (A \cap C))$

$= P(A) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]$

From (*), we get

$P(A \cap B^c \cap C^c) = P(A \cup B \cup C) - P(B) - P(C) + P(B \cap C)$

Question 4:

(a) A: {TTH, THT, HTT}

B: {TTH, THT, HTT, TTT}

C: {HHH, HTH, HHT, HTT}

D: {THH, TTH, THT, TTT}

(b) A^c: {HHH, THH, HTH, HHT, TTT}

A∪(C∩D): {TTH, THT, HTT}

A∩D^c: {HTT}

Question 5:

(a)

first entry \ second entry	P	a	b	c	d
a	0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	
b	$\frac{1}{12}$	0	$\frac{1}{12}$	$\frac{1}{12}$	
c	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{12}$	
d	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0	

(b) Denote the event "c is one of the chosen possibilities" by A

Thus A={(c,a),(c,b),(c,d),(a,c),(b,c),(d,c)}

P(A)=$\frac{1}{12}\times 6=\frac{1}{2}$

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