# Chapter 2
# Outcomes, Events and Probability

## 1 Summary

- – A *sample space* ($\Omega$) is a set of all the outcomes for a certain experiment.
  – A *event* is a subset of the sample space.
- – The *intersection* of event $A$ and event $B$ ($A \cap B$) is a set containing all the outcomes when both $A$ and $B$ occur.
  – The *union* of event $A$ and $B$ ($A \cup B$) is a set containing all the outcomes when $A$ or $B$ occurs.
  – The *complement* of event $A$, $A^c = \{\omega \in \Omega : \omega \notin A\}$, is a set containing all the outcomes when $A$ does not occur.
- – The event $A$ and event $B$ are *disjoint* or *mutually exclusive* if $A \cap B = \emptyset$.
  – The event $A$ implies the event $B$ if $A \subset B$.
- *DeMorgan's Laws*
  For any two events $A$ and $B$, we have
  – $(A \cup B)^c = A^c \cap B^c$ and
  – $(A \cap B)^c = A^c \cup B^c$.
- – A *probability function* P on a finite sample space $\Omega$ assigns to each event $A$ in $\Omega$ a number $P(A)$ in [0,1] such that
    * $P(\Omega) = 1$, and
    * $P(A \cup B) = P(A) + P(B)$ if A and B are disjoint.

    The number P(A) is called the probability that A occurs.
  – A probability function on an infinite (or finite) sample space $\Omega$ assings to each event $A$ in $\Omega$ a number $P(A)$ in [0,1] such that
    * $P(\Omega)$ = 1, and
    * $P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$ if $A_1, A_2, A_3, \ldots$ are disjoint events.
- – $P(A) = P(A \cap B) + P(A \cap B^c)$.
  – $P(A \cup B) = P(B) + P(A \cap B^c)$.
  – $P(A^c) = 1 - P(A)$.
- The probability of union.
  For any two events A and B we have
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
  For any three events A and B and C we have
  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.
- Products of sample spaces
  When we perform an experiment $n$ times, then the correponding sample space is $\Omega = \Omega_1 \times \Omega_2 \times \cdots \Omega_n$, where $\Omega_i$ for $i = 1, \ldots, n$ is a copy of the sample space of the original experiment. Moreover, the probability of the outcomes $(\omega_1, \omega_2, \ldots, \omega_n)$ is $P((\omega_1, \omega_2, \ldots, \omega_n)) = p_1 \cdot p_2 \cdots p_n$ if each $\omega_i$ has probability $p_i$.